Comments on: Cho, Lim, and Jang

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Princeton

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- How to repurpose the police estimate?

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- They then formulate estimates of p_0

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- Then they bootstrap to calibrate the precision of their $\hat{p}(0)$.

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- To do: derive sampling properties for $\hat{N}$

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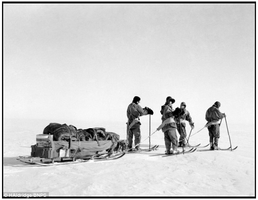
  - 

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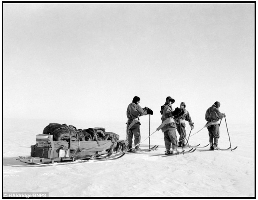
  - 
- Moral: Make full use of your options.

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- a useful (if not always welcome) reality check.

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- **Nice project!**